Math 100 ♦ ***Tangent Lines***

[#1] Determine the equation of the line secant to $y = x^2 - 4x$ as x–changes form –1 to 3.

(,) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $y - y_0 = m(x - x_0)$

(,)

[#2] Determine the equation of the line tangent to $y = x^2 - 4x$ at $x = 3$

(,) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $y - y_0 = m(x - x_0)$

(,)

Determine the slope of the tangent lines

[#3]
to $f(x) = x^2 - 2x - 1$ at (3, 2)

use tangent formula #1

$$m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

[#4]
to $f(x) = x^2 + 3x$ at (–1, –4)

use tangent formula #2

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

[#5]

to $f(x) = x^3 + 2$ at $(1, 3)$

use tangent formula #1

$$m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

[#6]

to $f(x) = 3x^2 - 2x$ at $(-1, 5)$

use tangent formula #2

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

[#7]

to $f(x) = \dfrac{4}{x - 2}$ at (3, 4)

use tangent formula #1

$$m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

[#8]

to $f(x) = \sqrt{x + 5}$ at (–1, 2)

use tangent formula #1

$$m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

[#9]

to $f(x) = \dfrac{1}{\sqrt{x-1}}$ at (2, 1)

use tangent formula #1

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

[#9]

to $f(x) = \dfrac{1}{\sqrt{x-1}}$ at (2, 1)

use tangent formula #2

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

[#1] Determine the equation of the line secant to $y = x^2 - 4x$ as x–changes form –1 to 3.

$(-1, 5)$

$(3, -3)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-3 - 5}{3 - -1} = \frac{-8}{4} = -2$

$y - y_0 = m(x - x_0)$

$y - 5 = -2(x + 1)$

$y = -2x + 3$

[#2] Determine the equation of the line tangent to $y = x^2 - 4x$ at $x = 3$

$(3, -3)$

$(3+h, (3+h)^2 - 4(3+h))$

$m = \frac{(3+h)^2 - 4(3+h) - -3}{3 + h - 3}$

$m = \frac{9 + 6h + h^2 - 12 - 4h + 3}{3 + h - 3}$

$m = \frac{h^2 + 2h}{h} = \frac{\cancel{h}(h+2)}{\cancel{h}}$

$m = 0 + 2 = 2$

$y + 3 = 2(x - 3)$

$y = 2x - 9$

Determine the slope of the tangent lines

[#3]

$f(x) = x^2 - 2x - 1$ at $(3, 2)$

a f(a)

use tangent formula #1

$$m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

$m = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$

$m = \lim_{h \to 0} \frac{(3+h)^2 - 2(3+h) - 1 - 2}{h}$

$m = \lim_{h \to 0} \frac{9 + 6h + h^2 - 6 - 2h - 1 - 2}{h}$

$m = \lim_{h \to 0} \frac{h^2 + 4h}{h}$

$m = \lim_{h \to 0} \frac{\cancel{h}(h+4)}{\cancel{h}} = 4$

[#4]

$f(x) = x^2 + 3x$ at $(-1, -2)$

use tangent formula #2

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$m = \lim_{x \to -1} \frac{f(x) - f(-1)}{x + 1}$

$m = \lim_{x \to -1} \frac{x^2 + 3x + 2}{x + 1}$

$m = \lim_{x \to -1} \frac{\cancel{(x+1)}(x+2)}{\cancel{x+1}} = 1$

$$a \qquad f(a)$$

[#5]

to $f(x) = x^3 + 2$ at (1, 3)

use tangent formula #1

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$m = \lim_{h \to 0} \frac{(1+h)^3 + 2 - 3}{h}$$

$$m = \lim_{h \to 0} \frac{h^3 + 3h^2 + 3h + 1 + 2 - 3}{h}$$

$$m = \lim_{h \to 0} \frac{\cancel{h}(h^2 + 3h + 3)}{\cancel{h}}$$

$$m = 0^2 + 3(0) + 3 = 3$$

EXPAND

$$(1+h)^3 = h^3 + 3h^2 + 3h + 1$$

[#6]

to $f(x) = 3x^2 - 2x$ at (–1, 5)

use tangent formula #2

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$m = \lim_{x \to -1} \frac{f(x) - f(-1)}{x + 1}$$

$$m = \lim_{x \to -1} \frac{3x^2 - 2x - 5}{x + 1}$$

$$m = \lim_{x \to -1} \frac{(3x-5)\cancel{(x+1)}}{\cancel{x+1}}$$

$$m = 3(-1) - 5 = -8$$

FACTOR

$$3x^2 - 2x - 5$$
$$3x^2 + 3x - 5x - 5$$
$$3x(x+1) - 5(x+1)$$
$$(x+1)(3x-5)$$

[#7]

to $f(x) = \frac{4}{x-2}$ at (3, 4)

use tangent formula #1

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$m = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$m = \lim_{h \to 0} \frac{\frac{4}{3+h-2} - 4}{h}$$

$$m = \lim_{h \to 0} \frac{\frac{4}{h+1} - 4}{h}$$

$$m = \lim_{h \to 0} \frac{4 - 4(h+1)}{h(h+1)}$$

$$m = \lim_{h \to 0} \frac{4 - 4h - 4}{h(h+1)}$$

$$m = \lim_{h \to 0} \frac{-4\cancel{h}}{\cancel{h}(h+1)}$$

$$m = -\frac{4}{0+1} = -4$$

[#8]

to $f(x) = \sqrt{x+5}$ at (−1, 2)

use tangent formula #1

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$m = \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h}$$

$$m = \lim_{h \to 0} \frac{\sqrt{-1+h+5} - 2}{h}$$

$$m = \lim_{h \to 0} \frac{\sqrt{h+4} - 2}{h} \cdot \frac{\sqrt{h+4} + 2}{\sqrt{h+4} + 2}$$

$$m = \lim_{h \to 0} \frac{h + 4 - 4}{h(\sqrt{h+4} + 2)}$$

$$m = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{h+4} + 2)}$$

$$m = \frac{1}{\sqrt{0+4} + 2} = \frac{1}{4}$$

$f(x) = \frac{1}{\sqrt{x-1}}$ at $(2, 1)$

$$m = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$m = \lim_{h \to 0} \frac{\frac{1}{\sqrt{2+h-1}} - 1}{h}$$

$$m = \lim_{h \to 0} \frac{\frac{1}{\sqrt{h+1}} - 1}{h}$$

multiply by LCD

$$m = \lim_{h \to 0} \frac{1 - \sqrt{h+1}}{h\sqrt{h+1}}$$

multiply by conjugate

$$m = \lim_{h \to 0} \frac{1 - (h+1)}{h\sqrt{h+1}(1+\sqrt{h+1})}$$

$$m = \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}\sqrt{h+1}(1+\sqrt{h+1})}$$

$$m = \frac{-1}{\sqrt{0+1}(1+\sqrt{0+1})}$$

$$m = \frac{-1}{2}$$

$$m = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

$$m = \lim_{x \to 2} \frac{\frac{1}{\sqrt{x-1}} - 1}{x - 2}$$

multiply by LCD

$$m = \lim_{x \to 2} \frac{1 - \sqrt{x-1}}{(x-2)\sqrt{x-1}}$$

multiply by conjugate

$$m = \lim_{x \to 2} \frac{1 - (x-1)}{(x-2)(\sqrt{x-1})(1+\sqrt{x-1})}$$

$$m = \lim_{x \to 2} \frac{\cancel{2-x}}{\cancel{(x-2)}(\sqrt{x-1})(1+\sqrt{x-1})}$$

$$m = \lim_{x \to 2} \frac{-1}{\sqrt{x-1}(1+\sqrt{x-1})}$$

$$m = \frac{-1}{1(2)} = -\frac{1}{2}$$