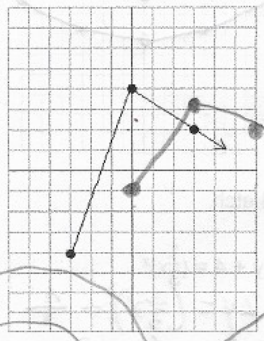
Math 12 • Transformations

Shown is y = f(x).
For each question, describe the transformations, sketch the graph, and write the mapping notation.

[#1]

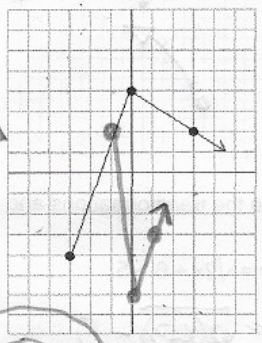


$y = \frac{1}{2}f(x - 3) + 1$

- VC by $\frac{1}{2}$
- HT 3 right
- VT 1 up

MAP $(x, y) \to (x+3, \frac{1}{2}y+1)$

[#2]

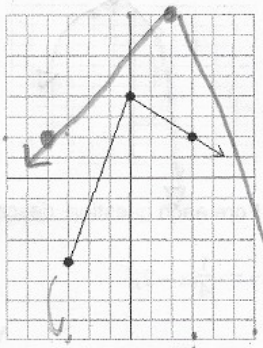


$y = -f(3x) - 2$

- VR
- HC by $\frac{1}{3}$
- VT 2 down

$(x, y) \to (\frac{x}{3}, -y-2)$

[#3]



$y = 2f(-\frac{1}{2}x + 1)$

$y = 2f(-\frac{1}{2}(x-2))$

- VE by 2
- HR
- HE by 2
- HT 2 right

$(x, y) \to (-2x+2, 2y)$

FACTOR INSIDE FUNCTION

MAPPING NOTATION

$(-3, -1) \to (0, -1)$

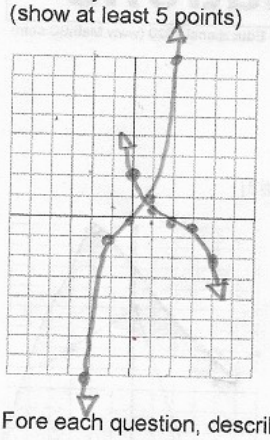
$(0, 4) \to (3, 3)$

$(3, 2) \to (6, 2)$

[#4]

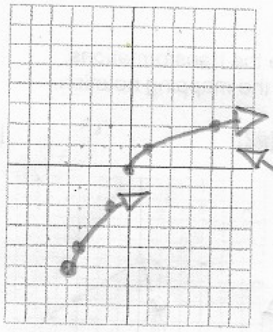
Sketch $y = x^3$
(show at least 5 points)

x	y
-2	-8
-1	-1
0	0
1	1
2	8



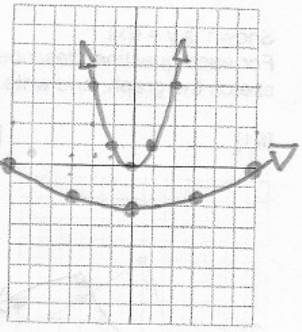
[#5]

Sketch $y = \sqrt{x}$
(show at least 3 points)



[#6]

Sketch $y = x^2$
(show at least 5 points)



Fore each question, describe the transformations and sketch.

$y = -\frac{1}{4}(x-2)^3$

VR

VC by $\frac{1}{4}$

HT 2 right

$y = \sqrt{2x+6} - 5$

$y = \sqrt{2(x+3)} - 5$

HC by $\frac{1}{2}$

HT 3 left

VT 5 down

$2y + 4 = (\frac{1}{3}x)^2$

$2y = (\frac{1}{3}x)^2 - 4$

$y = \frac{1}{2}(\frac{1}{3}x)^2 - 2$

VC by $\frac{1}{2}$

HE by 3

VT 2 down

abbreviations: $y = f(x)$

INSIDE (opp. op)
- HT = Horizontal Translation $y = f(x-2)$
- HE = Horizontal Expansion $y = f(\frac{1}{2}x)$
- HC = Horizontal Compression $y = f(2x)$
- HR = Horizontal Reflection $y = f(-x)$

OUTSIDE (as is)
- VT = Vertical Translation $y = f(x) + 2$
- VE = Vertical Expansion $y = 2f(x)$
- VC = Vertical Compression $y = \frac{1}{2}f(x)$
- VR = Vertical Reflection $y = -f(x)$
- I = Inverse

$x = f(y)$
or
$y = f^{-1}(x)$

Math 12 • Transformations

Write the new equation after the transformations have been applied

[#1] to y = f(x)
reflect in the y–axis
vertically expand by 3
translate 2 right, 6 down

HT

$y = 3f(-(x-2)) - 6$

WHEN HT FOLLOWS HR, HE, HC YOU NEED INSIDE BRACKETS

[#2] to y = g(x)
reflect in the x–axis
vertically compress by $\frac{3}{4}$
horizontally expand by 5
translate 6 left

$y = -\frac{3}{4}g\left(\frac{1}{5}(x+6)\right)$

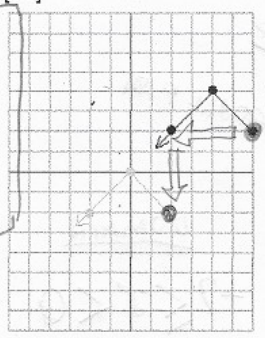
[#3] to y = x^3
horizontally compress by $\frac{2}{5}$
translate 3 right, 2 up

$y = \left(\frac{5}{2}(x-3)\right)^3 + 2$

y = f(x) is shown in black.
Write the equation of the graph shown in grey

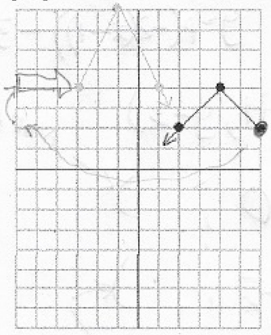
HR HE HC
VR VE VC
MUST BE DONE FIRST VISUALLY

[#4]



$y = f(x+4) - 4$

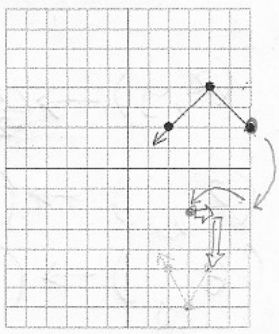
[#5]



$y = 2f(-(x-3))$

DO THESE BEFORE COUNTING OUT VT, HT

[#6]



$y = -f(2(x+1)) - 3$

[#7] The point (2, 5) is on y = f(x)
Find a point on y = –2f(x – 1) + 4

$(x, y) \rightarrow (x+1, -2y+4)$

$(2, 5) \rightarrow (3, -6)$

[#8] The point (–8, 4) is on y = g(x)
Find a point on $y = \frac{1}{2}f(4x + 12) - 1$

$y = \frac{1}{2}f(4(x+3)) - 1$

$(x, y) \rightarrow (\frac{x}{4} - 3, \frac{1}{2}y - 1)$

$(-8, 4) \rightarrow (-5, 1)$

NOTICE

[#9] The point (3, 0) is on y = 3h(2x) – 6
Find a point on y = h(x)

$(x, y) \rightarrow (\frac{x}{2}, 3y - 6)$

$(6, 2) \leftarrow (3, 0)$

ANSWER

$\frac{x}{2} = 3$
$x = 6$

$3y - 6 = 0$
$3y = 6$
$y = 2$

[#10] The domain of p(x) is $2 < x \leq 6$
Determine the domain of y = p(–2x + 2) – 3

$y = p(-2(x-1)) - 3$

$(x, y) \rightarrow (\frac{x}{-2} + 1, y - 3)$

NOT EQUAL PART

$(2, \ ) \rightarrow (0, \ )$

$(6, \ ) \rightarrow (-2, \ )$

DON'T CARE

EQUAL PART

DOMAIN

$-2 \leq x < 0$

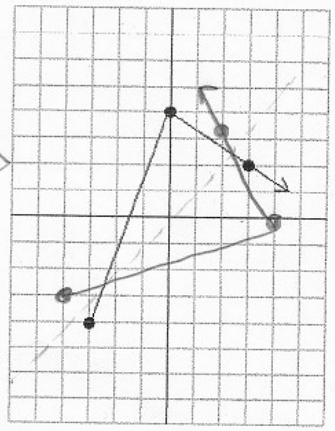
Math 12 • Transformations

switch x & y

[#1] Determine the inverse of the point (–7, 11)

(11, –7)

[#2] Shown is $y = f(x)$
Sketch $y = f^{-1}(x)$



Determine the inverse of each function

[#3] $y = 3x + 5$

$x = 3y + 5$

$x - 5 = 3y$

$\frac{x-5}{3} = y$

$y = \frac{1}{3}x - \frac{5}{3}$

[#4] $y = -\frac{3}{4}x + 2$

$x = -\frac{3}{4}y + 2$

$4x = -3y + 8$

$4x - 8 = -3y$

$\frac{4x-8}{-3} = y$

$y = -\frac{4}{3}x + \frac{8}{3}$

[#5] $y = \frac{2x-3}{x+2}$

$x = \frac{2y-3}{y+2}$

$x(y+2) = 2y - 3$

$xy + 2x = 2y - 3$

$2x + 3 = 2y - xy$

$2x + 3 = y(2 - x)$

$\frac{2x+3}{2-x} = y$

[#6] $y = (x-1)^2 - 2$

$x = (y-1)^2 - 2$

$x + 2 = (y-1)^2$

$\pm\sqrt{x+2} = y - 1$

$y = \pm\sqrt{x+2} + 1$

must include both arms

[#7] $y = \sqrt{x+2} - 1$ ← only has one arm

$x = \sqrt{y+2} - 1$

$x + 1 = \sqrt{y+2}$

$(x+1)^2 = y + 2$

$y = (x+1)^2 - 2;\ x \geq -1$

we only want one arm

[#8] The point (4, –5) is on $y = g(x)$
Find a point on $y = g^{-1}(x) - 2$

$(x, y) \rightarrow (y, x-2)$

$(4, -5) \rightarrow (-5, 2)$